**A. NUMBERS AND NUMBER SENSE**

**MIDDLE GRADES 5-8**

1. Use numbers in a variety of equivalent and interchangeable forms (e.g., integer, fraction, decimal, percent, exponential, and scientific notation) in problem-solving.
2. Demonstrate understanding of the relationships among the basic arithmetic operations on different types of numbers.
3. Apply concepts of ratios, proportions, percents, and number theory (e.g., primes, factors, and multiples) in practical and other mathematical situations.
4. Represent numerical relationships in graphs, tables, and charts.

EXAMPLES

* Use integers to write a play-by-play description of a game (e.g., football, soccer, or golf).
* Respond to the following in a journal: How can you get a smaller answer when you multiply? How can you get a larger answer when you divide?
* Estimate a 15% tip for a meal costing $38.60.

**SECONDARY GRADES**

1. Describe the structure of the real number system and identify its appropriate applications and limitations.
2. Explain what complex numbers (real and imaginary) mean and describe some of their many uses.

EXAMPLE

* Given two numbers such as 1/2 and 1/3, describe the real numbers between them.

**B. COMPUTATION**

**Students will understand and demonstrate computation skills. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Compute and model all four operations with whole numbers, fractions, decimals, sets of numbers, and percents, applying the proper order of operations.
2. Create, solve, and justify the solution for multi-step, real-life problems including those with ratio and proportion.

EXAMPLES

* Show that there must have been at least one misprint in a newspaper report on an election that reads:

- Yes votes 13,657 (42%)

- No votes 186,491 (58%)

and suggest two specific places a misprint may have occurred.

* On a number line, name the point located midway between 1/4 and 6.

**SECONDARY GRADES**

1. Use various techniques to approximate solutions, determine the reasonableness of answers, and justify the results.
2. Explain operations with number systems other than base ten.

EXAMPLE

* If 10% of U.S. citizens have a certain trait, and four out of five with the trait are men, determine what proportion of men have the trait and what proportion of women have the trait. Explain whether the answer depends on the proportion of U.S. citizens who are women, and if so, how?

**C. DATA ANALYSIS AND STATISTICS**

**Students will understand and apply concepts of data analysis. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Organize and analyze data using mean, median, mode, and range.
2. Assemble data and use matrices to formulate and solve problems.
3. Construct inferences and convincing arguments based on data.

EXAMPLES

* Conduct an experiment to determine the effects of fertilizer on plant growth, recording and analyzing information on charts and graphs.
* Using the height of students in the room, calculate the mean, median, mode, and range.

**SECONDARY GRADES**

1. Determine and evaluate the effect of variables on the results of data collection.
2. Predict and draw conclusions from charts, tables, and graphs that summarize data from practical situations.
3. Demonstrate an understanding of concepts of standard deviation and correlation and how they relate to data analysis.
4. Demonstrate an understanding of the idea of random sampling and recognition of its role in statistical claims and designs for data collection.
5. Revise studies to improve their validity (e.g., in terms of better sampling, better controls, or better data analysis techniques).

EXAMPLES

* Draw a scatterplot of the height of each student in the class vs. their shoe length and find the line of best fit using a graphics calculator or computer software.
* Design and conduct an experiment to estimate the population of clams in a given clam flat.

**D. PROBABILITY**

**Students will understand and apply concepts of probability. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Find the probability of simple events and make predictions by applying the theories of probability.
2. Explain the idea that probability can be represented as a fraction between and including zero and one.
3. Use simulations to estimate probabilities.
4. Find all possible combinations and arrangements involving a limited number of variables.

EXAMPLES

* Develop and analyze games of chance for a school carnival.
* Determine how many license plates are possible if the first two symbols are letters and the last four are numbers.

**SECONDARY GRADES**

1. Find the probability of compound events and make predictions by applying probability theory.
2. Create and interpret probability distributions.

EXAMPLE

* Determine the probability that a 90% free throw shooter will make exactly one of his/her upcoming two free throws.

**E. GEOMETRY**

**Students will understand and apply concepts from geometry. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Compare, classify, and draw two dimensional shapes and three dimensional figures.
2. Apply geometric properties to represent and solve real-life problems involving regular and irregular shapes.
3. Use a coordinate system to define and locate position.
4. Use the appropriate geometric tools and measurements to draw and construct two and three dimensional figures.

EXAMPLES

* Collect magazine pictures of different styles of architecture and identify all the geometric figures and relationships seen in each building.
* Display data with an accurately drawn and divided pie chart.

**SECONDARY GRADES**

1. Draw coordinate representations of geometric figures and their transformations.
2. Use inductive and deductive reasoning to explore and determine the properties of and relationships among geometric figures.
3. Apply trigonometry to problem situations involving triangles and periodic phenomena.

**F. MEASUREMENT**

**Students will understand and demonstrate measurement skills*.* Students will be able to:**

**MIDDLE GRADES 5-8**

1. Demonstrate the structure and use of systems of measurement.
2. Develop and use concepts that can be measured directly, or indirectly (e.g., the concept of rate).
3. Demonstrate an understanding of length, area, volume, and the corresponding units, square units, and cubic units of measure.

EXAMPLES

* Calculate the rate of speed of a moving object after measuring the distance traveled and the elapsed time.
* Examine areas that can be enclosed using 24 feet of fencing and figuring out the maximum area.
* Calculate the volume and surface areas of cones and pyramids.

**SECONDARY GRADES**

1. Use measurement tools and units appropriately and recognize limitations in the precision of the measurement tools.
2. Derive and use formulas for area, surface area, and volume of many types of figures.

EXAMPLES

* Discover and explore the distance formula using the Pythagorean Theorem.
* Using generalizations, compare the formula for the area of an n-sided, regular polygon to the formula for the area of a circle.

**G. PATTERNS, RELATIONS, FUNCTIONS**

**Students will understand that mathematics is the science of patterns, relationships, and functions. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Describe and represent relationships with tables, graphs, and equations.
2. Analyze relationships to explain how a change in one quantity can result in a change in another.
3. Use patterns and multiple representations to solve problems.

EXAMPLES

* Collect data on the cost of first class postage stamps for a one hundred year period of time and predict future costs for such stamps.
* Determine the units digit (ones' place) of (3)78.

**SECONDARY GRADES**

1. Create a graph to represent a real-life situation and draw inferences from it.
2. Translate and solve a real-life problem using symbolic language.
3. Model phenomena using a variety of functions (linear, quadratic, exponential, trigonometric, etc.).
4. Identify a variety of situations explained by the same type of function.

EXAMPLES

* Express the diameter of a circle as a function of its area and sketch a graph.
* Determine which of two ways of rolling a 8.5"x11" piece of paper into a cylinder gives the greater volume and whether there is a way to get even greater volume using a sheet of paper with the same area but different shape.

**H. ALGEBRA CONCEPTS**

**Students will understand and apply algebraic concepts. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Use the concepts of variables and expressions.
2. Solve linear equations using concrete, informal, and formal methods which apply the order of operations.
3. Analyze tables and graphs to identify properties and relationships in a practical context.
4. Use graphs to represent two-variable equations.
5. Demonstrate an understanding of inequalities and non-linear equations.
6. Find solutions for unknown quantities in linear equations and in simple equations and inequalities.

EXAMPLES

* Study the steepness of wheelchair ramps and stairs.
* Solve for x: 3x - 5 = 23 - x.

**SECONDARY GRADES**

1. Use tables, graphs, and spreadsheets to interpret expressions, equations, and inequalities.
2. Investigate concepts of variation by using equations, graphs, and data collection.
3. Formulate and solve equations and inequalities.
4. Analyze and explain situations using symbolic representations.

EXAMPLES

* Use measurements from shopping carts which are nested together to find a formula for the number of carts that will fit in a given space and a formula for the amount of space needed for a given number of carts.
* Solve the following problem: Given the formula for height of an object thrown upward with velocity v: h = ho + vt + (1/2) gt2, use quadratic functions and the quadratic formula to answer questions about the motion of projectiles and falling objects.

**I. DISCRETE MATHEMATICS**

**Students will understand and apply concepts in discrete mathematics. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Create and use networks to explain practical situations or solve problems.
2. Identify patterns in the world and express these patterns with rules.

EXAMPLE

* Use graphs and matrices to determine delivery routes from Augusta to other major cities in Maine with a combination of one way and round-trip routes.

**SECONDARY GRADES**

1. Use linear programming to find optimal solutions to a system.
2. Use networks to find solutions to problems.
3. Apply strategies from game theory to problem-solving situations.
4. Use matrices as tools to interpret and solve problems.

EXAMPLE

* Given a decreasing linear relationship between the selling price of a magazine and the number of people who will buy it, and given a fixed cost per copy that goes to production, analyze the profitability of the product and recommend a price range.

**J. MATHEMATICAL REASONING**

**Students will understand and apply concepts of mathematical reasoning. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Support reasoning by using models, known facts, properties, and relationships.
2. Demonstrate that multiple paths to a conclusion may exist.

EXAMPLE

* Prepare proposals for a fixed-height bridge and a draw bridge. Make recommendations after considering total cost, steepness of incline, traffic patterns, time of construction, etc.

**SECONDARY GRADES**

1. Analyze situations where more than one logical conclusion can be drawn from data presented.

EXAMPLE

* Given information about travel patterns in a local community, develop a convincing proposal for the logical placement of a bypass.

**K. MATHEMATICAL COMMUNICATION**

**Students will reflect upon and clarify their understanding of mathematical ideas and relationships. Students will be able to:**

**MIDDLE GRADES 5-8**

1. Translate relationships into algebraic notation.
2. Use statistics, tables, and graphs to communicate ideas and information in convincing presentations and analyze presentations of others for bias or deceptive presentation.

**SECONDARY GRADES**

1. Restate, create, and use definitions in mathematics to express understanding, classify figures, and determine the truth of a proposition or argument.
2. Read mathematical presentations of topics within the Learning Results with understanding.

EXAMPLES:

* Having read the definition of "kite", a student analyzes a collection of figures to decide which are kites. The student then proceeds to apply the kite definition to the families of quadrilaterals to determine which are kites and why.
* Student reads a manual or math text to successfully learn a new procedure.